## 5.0 Equations for uniform steady flow in open channel

### *5.1 Chezy's formula (1769 French engineer)*

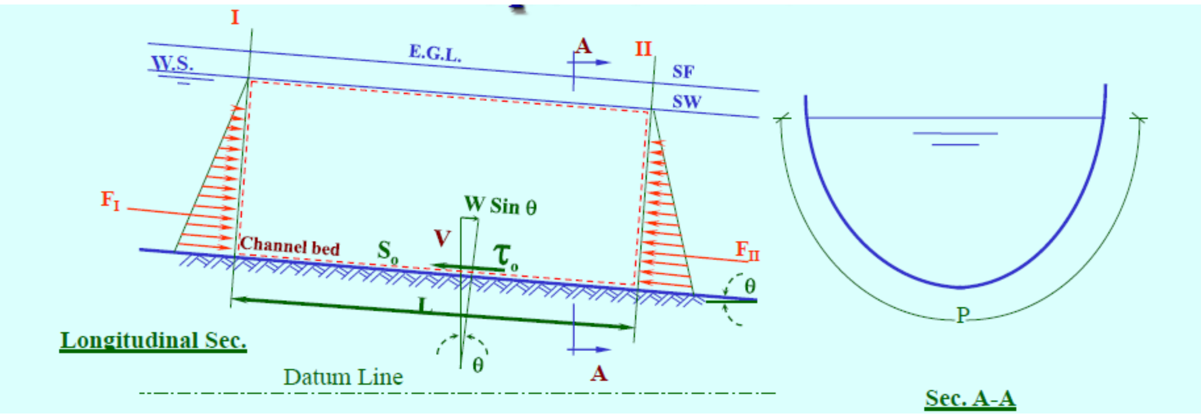



*Fig. (10) Force components in uniform open channel*

Consider the short reach of length L between station I and II, of a channel of steady uniform flow with cross section area A, and wetted perimeter P. The forces acting between the two sections (1 & 2) in the direction of the flow include: (1) the hydrostatic pressure forces, $F_1$, $F_2$, acting at each end of the free body; (2) the weight of the water body in the reach, W, which has a component, W sinθ, in the direction of the flow; and (3) the resistance force, $\tau_o$ PL. Applying the momentum equation between station I and II in the direction of motion:

$$\sum F = m\,a = 0 \quad \text{(for uniform flow } v = \text{constant).}$$

$$F_1 + W\sin\theta - F_2 - \tau_o\,PL = 0 \qquad (1)$$

For uniform flow $Y_1 = Y_2$ then $F_1 = F_2$

$$W\ \sin\theta \ = \tau_o\ PL \qquad (2)$$

$$\gamma AL\ \sin\theta = \tau_o PL$$

$$\sin\theta = \tan\theta = s \ \text{(for small } \theta)$$

$$\tau_o \ = \gamma \frac{A}{P} S = \gamma\ R\,S \qquad (3)$$

but $$\tau_o = C_f\ \rho\ \frac{v^2}{2} \quad \text{(as given before in pipe lines)} \qquad (4)$$

From equation (3) and (4)

$$\tau_o = \gamma\ R\,S\ = C_f\ \rho\ \frac{v^2}{2}$$

$$v\ =\sqrt{\frac{2\,\gamma}{C_f\rho}}\ \sqrt{R\,S}$$

$$v = C\sqrt{RS}$$

$$Q = CA\sqrt{RS} \quad \text{(Chezy's formula)}$$

where

Q = discharge ($L^3/T$)

V = mean velocity (L/T)

R = hydraulic radius = A/P (L)

S = slope of the bed, water surface, energy line (dimensionless)

C = Chezy's coefficient has the dimensions ($L^{1/2}/T$), (Resistance coefficient)

## *5.2 Determination of Chezy's coefficient of roughness (C).*

The roughness coefficient "C" depends on Reynolds Number of the flow, boundary roughness and shape of the channel cross section. Many attempts had been made to determine the value of Chezy's coefficient "C" as follows:

*A) The Darcy's formula*

$$h_f = \frac{f\,L\,v^2}{2gd} = \frac{f\,L\,v^2}{2g\,4R} \quad \text{where} \quad R = d/4$$

$$\frac{h_f}{L} = S = \frac{f\,v^2}{8gR}$$

$$v = \sqrt{\frac{8g}{f}}\sqrt{RS} \quad \rightarrow \text{Darcy}$$

$$\text{but} \quad v = C\sqrt{RS} \quad \rightarrow \text{Chezy}$$

$$\therefore \quad C = \sqrt{\frac{8g}{f}}$$

" f " can be found from the friction diagram or from the following equation:

$$f = a\,(1 + \frac{b}{R})$$

Where:

R = hydraulic radius;

a, b = constant depend on the boundary material

Table (1) Values of a & b

| Category | Very smooth Cement | Smooth Brick | Rough poor brick | Very rough earth |
|---|---|---|---|---|
| B | 0.03 | 0.07 | 0.25 | 1.25 |
| A | 0.00294 | 0.00373 | 0.0047 | 0.00549 |

*B) The Ganguillet and Kutter formula (1869).*

This formula expresses the Chezy's coefficient "C" in terms of S, R and coefficient of roughness "n".

$$C = \frac{23 + \left(0.00155/S\right) + \left(1/n\right)}{1 + \left(41.65 + \left(0.00281/S\right)\right)\dfrac{n}{\sqrt{R}}} \qquad \text{(Metric units)}$$

and

$$C = \frac{41.65 + \left(0.00281/S\right) + \left(1.811/n\right)}{1 + \left(41.65 + \left(0.00281/S\right)\right)\dfrac{n}{\sqrt{R}}} \qquad \text{(English units)}$$

*C) The Basin's formula (1897).*

The Basin's formula considers the Chezy's coefficient as function of R and coefficient of roughness "m" as follows:

$$C = 87 \Big/ \left(0.55 + m/\sqrt{R}\right) \qquad \text{(Metric units)}$$

and

$$C = 157.6 \Big/ \left(1 + m/\sqrt{R}\right) \qquad \text{(English units)}$$

where values of m represented in Table (2).

Table (2) values of m

| **Roughness category** | | **m** |
|---|---|---|
| Very smooth (cement) | | 0.061 |
| Smooth (brick) | | 0.116 |
| Rough (poor brick) | | 0.46 |
| Very rough (earth canal) | good | 0.85 |
| | ordinary | 1.3 |
| | bad | 1.75 |

*D) Manning equation (1890 Irish engineer).*

One of the best as well as one of the most widely used equation for open channel flow is Manning's equation. Manning found from many tests that the value of C varies approximately as $R^{1/6}$ , therefor, he observed that,

$$C = \frac{1}{n} R^{1/6}$$

Where; n = Manning's coefficient (depends on surface roughness $T/L^{1/3}$);

R = Hydraulic radius (L).

Therefore,

$$v = \frac{1}{n} R^{2/3} S^{1/2}$$

$$Q = \frac{1}{n} R^{2/3} S^{1/2} A \qquad \text{(Metric units)}$$

$$Q = \frac{1.49}{n} R^{2/3} S^{1/2} A \qquad \text{(English unit)}$$

Where: A = cross section area;

S = longitudinal slope.

Table (3) Values of Manning's Coefficient

| Channel material | Manning n |
|---|---|
| Earth canal | 0.025 |
| Lined canal (concrete) | 0.016 |
| Very weedy canal | 0.075 - 0.15 |
| Metal | 0.011 - 0.015 |
| Wood | 0.001 - 0.014 |

Practical value of n in Egypt

n = 0.025 for canals

n = 0.03 for drains

*E) Pavlovskii formula (proposed in 1925).*

$$C = \frac{1}{n} R^{y} \qquad \text{(Metric)}$$

$$y = 2.5\sqrt{n} - 0.13 - 0.75\sqrt{R}\left(\sqrt{n} - 0.1\right)$$

Where

C = Chezy's coefficient

n = Mannings coefficient

R = hydraulic radius

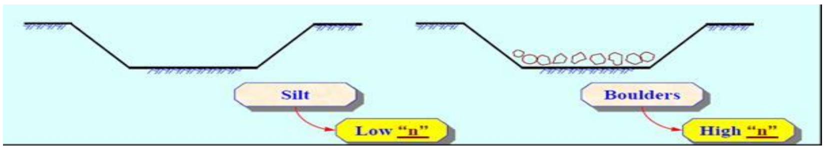
$$y = 1.5\sqrt{n} \qquad \text{for} \quad R < 1.0\ \text{m}$$

$$y = 1.3\sqrt{n} \qquad \text{for} \quad R > 1.0\ \text{m}$$

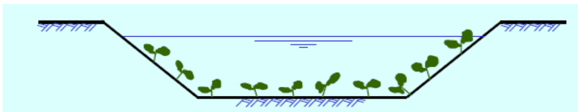
### *5.3 Factors affecting Manning's roughness coefficient*

The value of n is highly variable and depends on a number of factors:

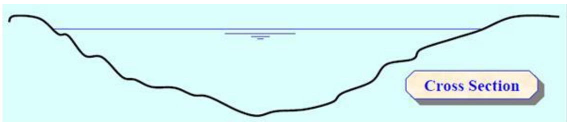
1) **Surface roughness:** is represented by the size and shape of the grains of the material forming the wetted perimeter and producing a retarding effect on the flow. The value of n is relatively low in fine grains and high in coarse grains.



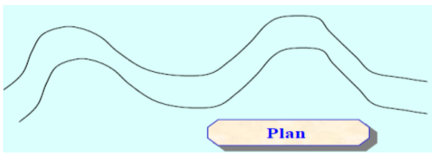

2) **Vegetation:** This effect depends mainly on height, density, distribution and type of vegetation.



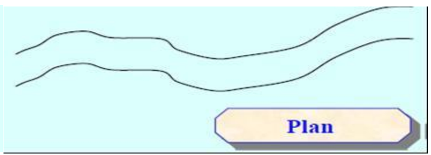
3) **Channel irregularity:** It comprises irregularities in wetted perimeter due to variations in cross section, size and shape along the channel length. Gradual and uniform changes will not affect the value of n.




4) **Channel alignment:** Smooth curvature will give low value of n.

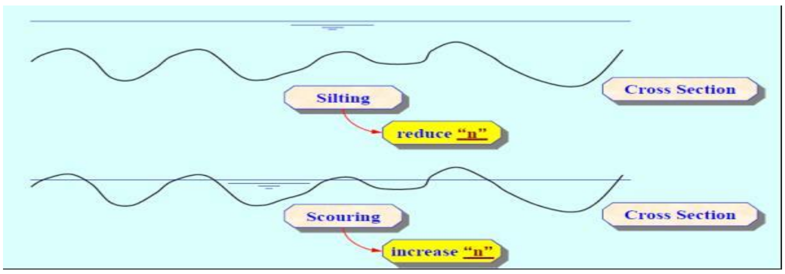



Low n

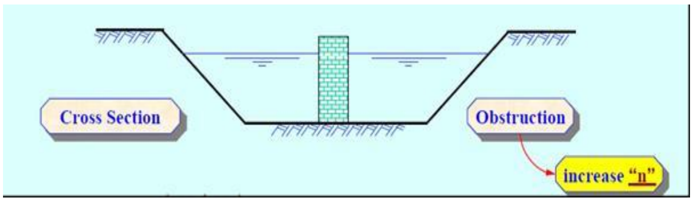



High n

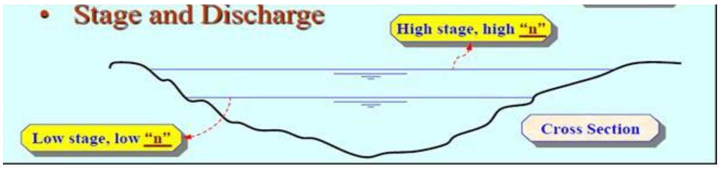
5) **Silting and scoring:** Silting may change a very irregular channel into a uniform and decreases n.



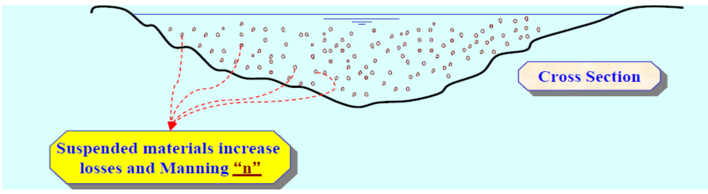
**6) Obstruction increases " n"**




**7) Stage and discharge:** The value of n increases with the increase of stage and discharge.



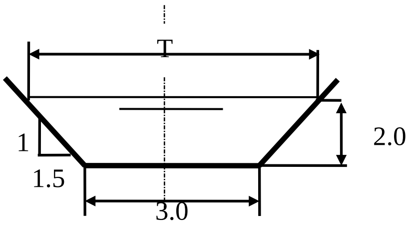

**8)** Suspended material: Increase the channel roughness.




*Example:*

A trapezoidal channel with a bottom width 3.0 m, side slope of 1.5:1, a longitudinal slope of 10 cm/km, and the water depth is 2.0 m. (1) Determine the discharge of the canal, if the canal is earth. (2) Determine the discharge if the canal is lined.

*Solution*



Trapezoidal

$A = by + zy^2 = 3*2 + 1.5*2^2 = 12.0\ m^2$

$P = b + 2y\sqrt{1+z^2} = 3 + 2*2\sqrt{1+1.5^2} = 10.2\ m$

$R = A/P = 12/10.2 = 1.18\ m$

$$Q = \frac{1}{n} R^{2/3} S^{1/2} A$$

$$Q = \frac{1}{0.025} (1.18)^{2/3} (10/10^5)^{1/2} (12) = 5.36\ m^3/sec \qquad \text{(Earth canal)}$$

$$Q = \frac{1}{0.016} (1.18)^{2/3} (10/10^5)^{1/2} (12) = 8.375\ m^3/sec \qquad \text{(Lined canal)}$$

*Example:*

A trapezoidal channel with a bottom width of 3.0 m, side slope of 1.5:1 a longitudinal slope of 10 cm/km and the discharge is 5.36 $m^3$/sec. (1) Determine the normal depth, if the coefficient of Manning is 0.025. (2) Determine the normal depth of the lined canal taking n = 0.016.

*Solution:*

$$Q = \frac{1}{n} R^{2/3} S^{1/2} A$$

$$A = by + zy^2 = 3y + 1.5y^2$$

$$P = b + 2y\sqrt{1+z^2} = 3 + 2y\sqrt{1+1.5^2} = 3+3.6y$$

$$R = A/P = \frac{3y + 1.5y^2}{3 + 3.6y}$$

$$Q = \frac{1}{0.025}\left(\frac{3y + 1.5y^2}{3 + 3.6y}\right)^{2/3} (10/10^5)^{1/2} (3y + 1.5y^2) = 5.36$$

From Manning equation

$$\frac{Q\,n}{S^{1/2}} = R^{2/3} A$$

$$\frac{5.36 * 0.025}{(10/10^5)^{1/2}} = 13.4$$

$$R^{2/3} A = \left(\frac{3y + 1.5y^2}{3 + 3.6y}\right)^{2/3} (3y + 1.5y^2) = 13.4$$

then y for earth canal = 2.0 m

y for lined canal = 1.65 m

* If the canal is lined ---- the depth of water decreases.